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RESILIENCE AND INOPERABILITY INDEXES FOR ELECTRICAL POWER SYSTEMS

Iony Patriota de Siqueira
CHESF
Brazil

SUMMARY

This paper describes a formal method to assess the resilience and inoperability of electric power systems using the results of contingency analysis. A model of disturbance propagation is proposed to avail their impact on resilience and inoperability indicators of Power Systems, and the contribution and cross impact from several market agents and actors during outages in the electric grid. The method is based on cascade disturbance propagation through the topology of the electric grid. Besides intrinsic forced and programmed outage rates, as well as protection reliability and dependability, time to manual or automatic reclosing are also modeled. Their joint contribution determines the final resilience and inoperability level of each grid node, and the fraction of responsibility of each agent. A working *MatLab*® implementation has been developed and documented. A simple case study will demonstrate its application.

KEYWORDS

Resilience, Inoperability, Risk, Power Quality, Performance.

1. Introduction

Current power system analysis comprises a large set of tools and techniques necessary to avail many aspects related to planning, operation, post-operation and maintenance. These include, for instance, models and tools for stability analysis, electromagnetic transients, optimal power flow, harmonic penetration, short-circuits, protection settings, reliability, etc. Every tool and model use overlapping sets of data, reports and formats, while some integrated tools concentrate several techniques in a centralized model. Beside such diversity of tools and problems, system analysts, operators, owners, investors, regulators and other stakeholders still need additional summary information about the robustness of current and planned grid topologies, to avail the impact of system outages. Part of this information can be provided by measurements and estimation of resilience and inoperability of electric power systems.

The concept of resilience refers to the capacity of a system to return to its natural or normal state, after an emergency situation. Inoperability refers to the difficulty faced by a system to recover from an emergency situation. Both concepts can be measured by the capacity and velocity a system has in recovering after the inception of forced outage on its components.

The main contribution of this paper is a proposal of definitions and formal methods to evaluate expected values of power system resilience and inoperability indexes, and the contribution and cross impact of several agents and market players in their values. The method is based on data generated by current power system analysis programs, used to build graph models of forced and planned outages, and their cascade propagation through the topology of the electrical grid. Besides primary equipment outage and maintenance rates, other issues can be modeled like security and dependability of protection, breaker failures, and operator time to restoration or automatic reclosing after a disturbance. Their joint contribution determines the overall level of resilience and inoperability at each network node, their subsystems or the complete power network, and the portion of responsibility of each player. The models are based on reachability matrices for cascading outages, adequate to avail the impact of disturbances, maintenance, operating and planning actions from each player. A working *MatLab®* implementation has been developed as part of a research project in power system analysis.

The next section of this paper defines the concepts and indexes related to resilience and inoperability. This is followed in section three by a vector representation of forced and programmed outage of components in a power grid, including protection reliability and dependability, connected loads and outage duration, partitioned among asset owners. A small, multi-company power system is used throughout the paper, to illustrate the approach. The fourth section introduces adjacency and reachability matrices to model the topology and propagation of outage events on the power grid. Graph theoretic concepts are used to support these models. The fifth and sixth sections use these matrices to estimate interruption frequencies and duration for each network load. Resilience and inoperability indexes are estimated for each grid point in section seven, and for sub networks in section eight, partitioned among network players. Section nine presents a simple case study using the *MatLab®* program. The conclusions in section ten summarize the features of the model.

2. Definitions

Resilience is a concept that refers to the capacity of a system to return to its natural or normal state, recovering from an unplanned situation. Inoperability refers to the recovering difficulty a system has to overcome a contingency. For the objectives of this paper, these terms will be used to denote respectively the recovering capacity and speed of a power system, after the occurrence of forced interruptions in its components. The following definitions and performance indexes will be introduced to measure these aspects:

Component Resilience - The Component Resilience (RS) of a power system item is defined by the fraction of its forced outages where it is possible to immediately return it to operation after an interruption, by automation or operational action, independent of the correction of failure on the network that originated it, being estimated by:

$$RS_i = \frac{\sum ForcedOutageOfComponent(i)WithImediateRestoration}{\sum ForcedOutageOfComponent(i)}, \qquad (1)$$

where the summations extend to all forced outages of component *i* in the evaluation period.

Component Inoperability - The Component Inoperability (IN) of a power system item is defined by the fraction of time each component is unavailable from forced outages, with no immediate restoration after an interruption, by automation or operational action, before the correction of failure that originated the outage, in relation to the total time of its forced unavailability, being estimated by:

$$IN_i = \frac{\sum ForcedUnavailabilityOfComponent(i)WithoutImediateRestoration}{\sum ForcedUnavailabilityOfComponent(i)}, \qquad (2)$$

where the summations extend to all forced outages of component *i* in the evaluation period.

These indexes can be extended to evaluate the resilience and inoperability of subsystems in a power network, using the following definitions:

System Resilience - The System Resilience (RSS) of a power system sub network is defined by the fraction of accumulated forced outages of its components that are possible to be immediately restored after an interruption, by automation or operational action, independent of the correction of failure on the network that originated it, being estimated by:

$$RS_{Si} = \frac{\sum_{j \in i} ForcedOutageOnSystemItem(j)WithImediateRestoration}{\sum_{j \in i} ForcedOutageOnSystemItem(j)}, \qquad (3)$$

where the summations extend to all forced outages of items from subsystem *i* in the evaluation period.

System Inoperability - The System Inoperability (INS) of a power system is defined by the fraction of accumulated time the components of a subsystem are unavailable from forced outages, with no immediate restoration after an interruption, by automation or operational action, before the correction of the originating failure, in relation to the total time of forced unavailability of the network components, being estimated by:

$$IN_{Si} = \frac{\sum_{j \in i} ForcedUnavailabilityOnSystemItem(j)WithoutImediateRestoration}{\sum_{j \in i} ForcedUnavailabilityOnSystemItem(j)}, \qquad (4)$$

where the summations extend to all unavailability of items from subsystem *i* in the evaluation period.

These definitions allow the post-operation evaluation of these indexes, based on the historical data gathered during each forced outage. To allow their pre-operation estimation, a model will be defined based on component and network modeling based on the reachability of forced outages.

3. Component Modeling

Each component on a power grid has a behavior that depends on its intrinsic reliability, planned outages, protection, operation and connected load or generation. For the objectives of this paper, a component is any smallest (high voltage) part of the power system that can be isolated from the grid, by disconnecting means. This includes bus bars, transformers, lines, generators, etc., but not instrument transformers, breakers, switches, etc. The former are main functions of the power systems, while the later are considered part of these functions. The term **intrinsic** refer to its own characteristic, independent of the rest of the grid. These aspects can be modeled by vectors listing their intrinsic parameters, partitioned by the *N* connected agents on the grid, as shown on expressions (5) to (10), where *n* is the number of grid components. Forced outage duration and frequency (equations (5) and (9)) follow the traditional meaning of these terms in Power Systems. Protection reliability (equation (6)) is the mean probability that the protection of an item will trip it for an internal fault, while the protection vulnerability (equation (7)) is the mean probability that the protection of an item will trip for an external fault within its reaching zone. Time to restore (equation (8)) denotes the mean time an item stays unconnected, after an outage, for operating reasons. Vector equation (10) models the external load (positive) or generation (negative load) connected to each network component. All these data are usually available from standard Asset Management Systems operated by utilities.

Intrinsic Forced Outage Frequency $$\mathbf{f}_I = [f_{I1} \quad f_{I2} \quad \dots \quad f_{In}]^T \tag{5}$$

Intrinsic Protection Reliability $$\mathbf{C} = [C_1 \quad C_2 \quad \dots \quad C_n]^T \tag{6}$$

Intrinsic Protective Vulnerability $$\mathbf{V} = [V_1 \quad V_2 \quad \dots \quad V_n]^T \tag{7}$$

Intrinsic Time to Restore $$\mathbf{r} = [r_1 \quad r_2 \quad \dots \quad r_n]^T \tag{8}$$

Intrinsic Forced Outage Duration $$\mathbf{d}_I = [d_{I1} \quad d_{I2} \quad \dots \quad d_{In}]^T \tag{9}$$

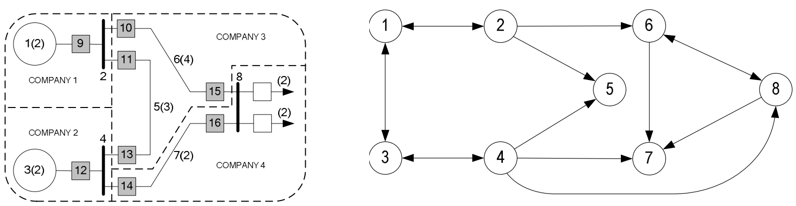
Connected Load $$\mathbf{l} = [l_1 \quad l_2 \quad \dots \quad l_n]^T \tag{10}$$

4. Network Modeling

Functional dependency among equipment in a power grid can be modeled by an adjacency matrix that connects those items whose forced outages are related. The concept of forced adjacency applies to radially connected items, to items located on the same protection zone, but also to distinct zones tripped by overloads, faults, under and over voltages, or remote zones tripped by load or wide area protection systems. For any grid a Forced Adjacency Matrix **I** can be defined by the expression:

$$\mathbf{I} = [I_{ij}] = p_i I p_j = \begin{cases} 1, & \text{if } p_i \text{ trips } p_j \text{ in a forced outage} \\ 0, & \text{otherwise,} \end{cases} \tag{11}$$

where p_i e p_j ($i, j \leq n$) are components of the grid, as exemplified by the graph of Fig. 1, shown for a typical power grid, whose item capacities are indicated in parentheses. Generation companies 1 and 2, transmission company 3 and distribution company 4, are shown separated by dotted lines. The graph models all functional dependencies during forced outages of related components, obtained by contingency studies. The contingency analysis is based on a reference power flow case where all generators (4 pu of capacity) are necessary to attend the 4 pu loading on bus 8, with a 2 pu load flow on each line 6 and 7. A contingency in line 6 (4 pu of capacity), will overload line 7 (2 pu of capacity) if it trips unexpectedly, but not if it is taken out in a planned way. Line 5 (3 pu of capacity) is operated with no load in the reference case.



$$\mathbf{I} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Figure 1: Graph and Forced Adjacency Matrix of Typical Electrical Power Grid

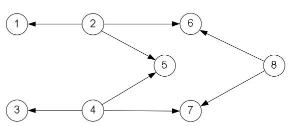
Likewise, it is possible to model the operational dependency among items by relating those whose planned outage will always occur at the same time. It applies, for instance, to transmission lines and transformers with their breakers, components that overload with the outage of other elements, items on the same protection zone, distinct items tripped to avoid overloads, or remote items to avoid operation of load sharing schemes, items on radial systems, etc. For any grid a Planned (Outage) Adjacency Matrix P can also be defined by:

$$\mathbf{P} = \left[P_{ij}\right] = p_i P p_j = \begin{cases} 1, & \textit{if } p_i \textit{ trips } p_j \textit{ in a planned outage} \\ 0, & \textit{otherwise,} \end{cases} \tag{12}$$

where p_i and p_j $(i, j \leq n)$ are components of the grid, as exemplified by the graph and associated matrix of Fig. 3. The more meshed the grid, the sparser are matrices **P** and **I**. In the same way, the operational dependency among protection systems, breakers and protected components can be modeled relating those items whose faults are detected by each protection or affected by breaker trips. It applies, for instance, to items located on the same protection zone, or on adjacent zones at the reach of the protection, when it acts as a backup protection. For any grid it is possible to define a Protective Adjacency Matrix **T**, by the expression

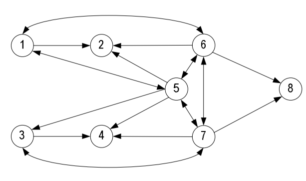
$$\mathbf{T} = \left[T_{ij}\right] = p_i T p_j = \begin{cases} 1, & \textit{if } p_i\textit{´s protection protects } p_j \\ 0, & \textit{otherwise,} \end{cases} \tag{13}$$

exemplified by the graph and matrix of Fig. 2.



Planned Adjacency Graph

$$\mathbf{P} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$



Protective Adjacency Graph

$$\mathbf{T} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 2: Graphs of Planned and Protective Adjacency Matrices

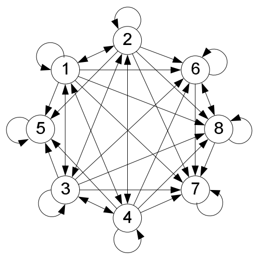
These three Boolean matrices, **P**, **I** e **T**, model the grid topology for the propagation of planned and forced events. To cascade the reach of every outage it is necessary to develop Forced and Planned Reachability Matrices ($\mathbf{A}_I$ e $\mathbf{A}_P$), and a Protective Vulnerability Matrix ($\mathbf{T}_C$) by the following operations (Boolean for $\mathbf{A}_I$ and $\mathbf{A}_P$, and algebraic for $\mathbf{T}_C$) in equations (14)(15)(16):

$$\mathbf{A}_I = (\mathbf{I} + \mathbf{U})^r = (\mathbf{I} + \mathbf{U})^{r-1} \neq (\mathbf{I} + \mathbf{U})^{r-2}, \tag{14}$$

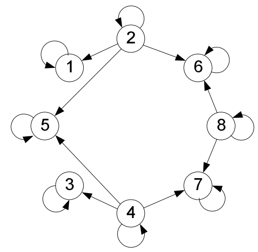
$$\mathbf{A}_P = (\mathbf{P} + \mathbf{U})^r = (\mathbf{P} + \mathbf{U})^{r-1} \neq (\mathbf{P} + \mathbf{U})^{r-2}, \tag{15}$$

$$\mathbf{T}_C = \mathbf{C}_D (\mathbf{T} - \mathbf{C}_D.\mathbf{T})^T + \mathbf{V}_D \mathbf{T}, \tag{16}$$

where the subscript D denotes the diagonal matrix, and r (the smallest positive integer that satisfies the above equations) is the maximum extension of (forced or planned) cascading outages originated from any grid item, and $\mathbf{U}$ is the unit diagonal matrix. They link all items that must be tripped together, following the outage of one of them. In equation (16), the parcels ($\mathbf{T}$-$\mathbf{C}_D\mathbf{T}$) e ($\mathbf{V}_D\mathbf{T}$) are stochastic matrices of refusal or wrong trip chances from protection of component i, or breaker, for a fault in j. They give the probability of tripping of each grid element for a protection or breaker failure in other items, as a function of the Protective Adjacency Matrix **T.** Figure 3 shows the resultant graphs and reachability matrices for the above example.



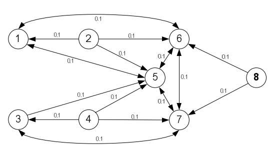
$$\mathbf{A}_I = \begin{array}{cccccccc|c} 1&2&3&4&5&6&7&8& \\ \hline 1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&2 \\ 1&1&1&1&1&1&1&1&3 \\ 1&1&1&1&1&1&1&1&4 \\ 0&0&0&0&1&0&0&0&5 \\ 0&0&0&0&0&1&1&1&6 \\ 0&0&0&0&0&0&1&0&7 \\ 0&0&0&0&0&1&1&1&8 \end{array}$$



$$\mathbf{A}_P = \begin{array}{cccccccc|c} 1&2&3&4&5&6&7&8& \\ \hline 1&0&0&0&0&0&0&0&1 \\ 1&1&0&0&1&1&0&0&2 \\ 0&0&1&0&0&0&0&0&3 \\ 0&0&1&1&1&0&1&0&4 \\ 0&0&0&0&1&0&0&0&5 \\ 0&0&0&0&0&1&0&0&6 \\ 0&0&0&0&0&0&1&0&7 \\ 0&0&0&0&0&1&1&1&8 \end{array}$$

Figure 3: Graphs of Forced and Planned Reachability Matrices

Figure 4 illustrates the graph and Protective Vulnerability Matrix $\mathbf{T}_\mathbf{C}$ for the example, with a typical protection reliability of 90%, and 1% probability of refusal on demand. In interconnected power systems, with N players, the elements of matrices $\mathbf{A}_I$**,** $\mathbf{A}_P$ e $\mathbf{T}_C$ can be partitioned by system (or company, or control area) Interconnection Reachability Matrices ($\mathbf{A}_{Iij}$, $\mathbf{A}_{Pij}$ e $\mathbf{T}_{Cij}$) among players, which avail the impacts of forced and planned outages, including from protection or breaker failure, originated from player i over player j. Figures 1 to 4 show these partitioning among players 1 to 4, by dividing lines in the matrices. Matrices $\mathbf{A}_I$ e $\mathbf{A}_P$, cascade the consequences of each outage on grid topology. The difference of these matrices, ($\mathbf{A}_I$-$\mathbf{A}_P$), defines all items that can be immediately reenergized, without waiting for the restoration of the faulted item that originated the outage. If implemented in an energy management system, it could help on the system restoration after a major blackout. Matrix $\mathbf{A}_\mathrm{P}$ also defines the items that must wait the restoration of a faulted component, before they can be returned to operation.



$$\mathbf{T}_C = \begin{array}{cccccccc|c} L1.&2.&3.&4.&5.&6.&7.&8.& \\ \hline 0&0&0&0&0.1&0.1&0&0&1 \\ 0.1&0&0&0&0.1&0.1&0&0&2 \\ 0&0&0&0&0.1&0&0.1&0&3 \\ 0&0&0.1&0&0.1&0&0.1&0&4 \\ 0.1&0&0.1&0&0&0.1&0.1&0&5 \\ 0.1&0&0&0&0&0&0.1&0&6 \\ 0&0&0.1&0&0&0.1&0&0&7 \\ 0&0&0&0&0&0.1&0.1&0&8 \end{array}$$

Figure 4: Protective Vulnerability Graph and Matrix

5. Interruption Frequencies

From these parameters, it is possible to avail the vectors with Total Forced and Planned Outage Frequencies ($\mathbf{F}_I$ and $\mathbf{F}_P$) and Total Outage Frequency from Protection Failure ($\mathbf{F}_C$) for all components, partitioned by N players in equations (17)(18)(19):

$$\mathbf{F}_I = \left[F_{Ii}\right] = \mathbf{A}_I^T \mathbf{f}_I\,, \qquad i \le n\,, \tag{17}$$

$$\mathbf{F}_P = \left[F_{Pi}\right] = \mathbf{A}_P^T \mathbf{f}_P\,, \qquad i \le n\,, \tag{18}$$

$$\mathbf{F}_C = \left[F_{Ci}\right] = \mathbf{T}_C \mathbf{f}_I\,, \qquad i \le n\,, \tag{19}$$

giving the expected frequencies of forced and planned outage of each item, from intrinsic causes, or originated in other grid component or protection/breaker failure.

6. Interruption Duration

Interruption duration of each grid element results from the combined frequency and duration of forced and planned outages, and restoration times, measured by the vectors of Total Forced Outage Duration ($\mathbf{D}_{AI}$), Total Planned Outage Duration ($\mathbf{D}_{AP}$), Total Time to Restore after a Forced Outage ($\mathbf{R}_{AI}$), Total Time to Restore after a Planned Outage ($\mathbf{R}_{AP}$), and Total Outage Duration due to Protection Failure ($\mathbf{D}_{AC}$), partitioned by N players respectively in equations (20)(21)(22)(23)(24):

$$\mathbf{D}_{AI} = [D_{AIi}] = \mathbf{A}_I^T * (\mathbf{d}_I .* \mathbf{f}_I), \qquad i \le n, \tag{20}$$

$$\mathbf{D}_{AP} = [D_{APi}] = \mathbf{A}_P^T * (\mathbf{d}_P .* \mathbf{f}_P), \qquad i \le n, \tag{21}$$

$$\mathbf{R}_{AI} = [R_{AIi}] = \mathbf{A}_I^T * (\mathbf{r} .* \mathbf{f}_I), \qquad i \le n, \tag{22}$$

$$\mathbf{R}_{AP} = [R_{APi}] = \mathbf{A}_P^T * (\mathbf{r} .* \mathbf{f}_P), \qquad i \le n, \tag{23}$$

$$\mathbf{D}_{AC} = [D_{ACi}] = \mathbf{T}_C * (\mathbf{r} .* \mathbf{f}_I), \qquad i \le n. \tag{24}$$

The component resilience and inoperability of each item can now be estimated from these data and their definitions.

7. Component Resilience and Inoperability

The estimation of these indexes will consider that the total frequency of forced outage of each component will be given by the sum of the Total Forced and Planned Outage Frequencies ($\mathbf{F}_I$) and Total Outage Frequency from Protection Failure ($\mathbf{F}_C$) originated on the network that impact each component, estimated by the vector ($\mathbf{F}_I$+$\mathbf{F}_C$). Some outages can be recovered immediately, before even the correction of the originating failure, being determined by simulating the planned propagation of outages from the failed components, estimated by the vector ($\mathbf{A}_P{}^T$*$\mathbf{f}_I$), applying the Planned Reachability Matrix ($\mathbf{A}_P$) to the Intrinsic Forced Outage Frequency ($\mathbf{f}_I$). The relation between these factors represent the fraction of forced outages that in principle can be recovered before the correction of the originated failure as they represent planned outages starting from the failed component. So, the vector of Component Resilience (**RS**) of all network items can be estimated by:

$$\mathbf{RS} = (\mathbf{A}_P^T * \mathbf{f}_I)./(\mathbf{F}_I + \mathbf{F}_C). \tag{25}$$

In the same way, the cumulative duration of forced outages of each item, in an evaluation period, will be given by the sum of Total Forced Outage Duration ($\mathbf{D}_{AI}$), Total Time to Restore after a Forced Outage ($\mathbf{R}_{AI}$) and Total Outage Duration due to Protection Failure ($\mathbf{D}_{AC}$) of each element, estimated by the vector ($\mathbf{D}_{AI}$+$\mathbf{R}_{AI}$+$\mathbf{D}_{AC}$). Part of these outages will be immediately recovered, even before the correction of the originating fault, by the same reasons as before, with intervals determined by the planned propagation of the recovering times starting from the faulted components, estimated by the vector $\mathbf{A}_P{}^T$*($\mathbf{r}$.*$\mathbf{f}_I$). The complement of the relation between these factors represents the fraction of outage time that in principle could not be recovered before the correction of the originating fault. So, a vector with the Inoperability (**IN**) of all network items can be estimated by:

$$\mathbf{IN} = \mathbf{1} - [\mathbf{A}_P^T * (\mathbf{r} .* \mathbf{f}_I)]./(\mathbf{D}_{AI} + \mathbf{R}_{AI} + \mathbf{D}_{AC}). \tag{26}$$

The estimation of the corresponding system resilience and inoperability follows from their definition.

8. System Resilience and Inoperability

To avail equivalent system resilience and inoperability, a System Matrix **S** will be defined to model the ownership of each component by a system (or control area) using the following expression:

$$\mathbf{S} = [S_{ij}] = s_i Sp_j = \begin{cases} 1, & \textit{if component } p_j \textit{ is owned by system } s_i \\ 0, & \textit{otherwise.} \end{cases} \tag{27}$$

System Resilience and Interoperability can now be estimated considering only outages that impact each system, applying the System Matrix **S** to the numerators and denominators of fractions (25) e (26), resulting in:

$$\mathbf{RS}_S = [\mathbf{S} * (\mathbf{A}_P^T * \mathbf{f}_I)]./[\mathbf{S} * (\mathbf{F}_I + \mathbf{F}_C)], \tag{28}$$

$$\mathbf{IN}_S = \mathbf{1} - \{\mathbf{S} * [\mathbf{A}_P^T * (\mathbf{r}.*\mathbf{f}_I)]\}./[\mathbf{S} * (\mathbf{D}_{AI} + \mathbf{R}_{AI} + \mathbf{D}_{AC})]. \tag{29}$$

Scalar values for the Resilience (RSR) and Inoperability (INR) for the whole network can be estimated by replacing the System Matrix **S** by the unit vector of same dimension of the network in equations (28) and (29):

$$RS_R = [ones(1,n) * (\mathbf{A}_P^T * \mathbf{f}_I)]./[ones(1,n) * (\mathbf{F}_I + \mathbf{F}_C)], \tag{30}$$

$$IN_R = 1 - \{ones(1,n) * [\mathbf{A}_P^T * (\mathbf{r}.*\mathbf{f}_I)]\}./[ones(1,n) * (\mathbf{D}_{AI} + \mathbf{R}_{AI} + \mathbf{D}_{AC})]. \tag{31}$$

It is also possible to evaluate the resilience and inoperability of the consumers and generators of the network, limiting the estimation only to the load and generation points, pondering each outage by the signal of interrupted power. The following expressions result for the resilience and inoperability of the load points and corresponding sub networks:

$$\mathbf{RS}_L = [(\mathbf{A}_P^T * \mathbf{f}_I).*(\mathbf{l}.*(\mathbf{l} > \mathbf{0}))]./[(\mathbf{F}_I + \mathbf{F}_C).*(\mathbf{l}.*(\mathbf{l} > \mathbf{0}))], \tag{32}$$

$$\mathbf{IN}_L = \mathbf{1} - [(\mathbf{A}_P^T * (\mathbf{r}.*\mathbf{f}_I)).*(\mathbf{l}.*(\mathbf{l} > \mathbf{0}))]./[(\mathbf{D}_{AI} + \mathbf{R}_{AI} + \mathbf{D}_{AC}).*(\mathbf{l}.*(\mathbf{l} > \mathbf{0}))], \tag{33}$$

$$\mathbf{RS}_{SL} = [\mathbf{S} * ((\mathbf{A}_P^T * \mathbf{f}_I).*(\mathbf{l}.*(\mathbf{l} > \mathbf{0})))]./[\mathbf{S} * ((\mathbf{F}_I + \mathbf{F}_C).*(\mathbf{l}.*(\mathbf{l} > \mathbf{0})))], \tag{34}$$

$$\mathbf{IN}_{SL} = \mathbf{1} - \{\mathbf{S} * [(\mathbf{A}_P^T * (\mathbf{r}.*\mathbf{f}_I)).*(\mathbf{l}.*(\mathbf{l} > \mathbf{0}))]\}./[\mathbf{S} * ((\mathbf{D}_{AI} + \mathbf{R}_{AI} + \mathbf{D}_{AC}).*(\mathbf{l}.*(\mathbf{l} > \mathbf{0})))], \tag{35}$$

where the subscript L refers to the load demanded by each network point. Similar expressions result for the resilience and inoperability of generation points, by changing the signal ">"of the power flow to "<" on the above equations, resulting in:

$$\mathbf{RS}_G = [(\mathbf{A}_P^T * \mathbf{f}_I).*(\mathbf{l}.*(\mathbf{l} < \mathbf{0}))]./[(\mathbf{F}_I + \mathbf{F}_C).*(\mathbf{l}.*(\mathbf{l} < \mathbf{0}))], \tag{36}$$

$$\mathbf{IN}_G = \mathbf{1} - [(\mathbf{A}_P^T * (\mathbf{r}.*\mathbf{f}_I)).*(\mathbf{l}.*(\mathbf{l} < \mathbf{0}))]./(\mathbf{D}_{AI} + \mathbf{R}_{AI} + \mathbf{D}_{AC}).*(\mathbf{l}.*(\mathbf{l} < \mathbf{0}))], \tag{37}$$

$$\mathbf{RS}_{SG} = [\mathbf{S} * ((\mathbf{A}_P^T * \mathbf{f}_I).*(\mathbf{l}.*(\mathbf{l} < \mathbf{0})))]./[\mathbf{S} * ((\mathbf{F}_I + \mathbf{F}_C).*(\mathbf{l}.*(\mathbf{l} < \mathbf{0})))], \tag{38}$$

$$\mathbf{IN}_{SG} = \mathbf{1} - \{\mathbf{S} * [(\mathbf{A}_P^T * (\mathbf{r}.*\mathbf{f}_I)).*(\mathbf{l}.*(\mathbf{l} < \mathbf{0}))]\}./[\mathbf{S} * ((\mathbf{D}_{AI} + \mathbf{R}_{AI} + \mathbf{D}_{AC}).*(\mathbf{l}.*(\mathbf{l} < \mathbf{0})))], \tag{39}$$

where the subscript G refers to the power generated at each point or sub network. The following example shows these calculi for a system with 4 sub networks or control areas.

9. Case Study

Table 1 shows the values of Resilience and Inoperability of the components of the network of Fig. 1, and its interconnected subsystems, using a *MatLab®* computer program that implements equations (25) to (39). Figure 5 shows the percent Resilience and Inoperability of the components in a one line diagram.

Table 1 – Resilience and Inoperability

Company	Company 1		Company 2		Company 3		Company 4	
Item	1	2	3	4	5	6	7	8
$\mathbf{RS}^T =$	0.5714	0.4286	0.4286	0.1429	0.5022	0.7740	0.5525	0.3684
$\mathbf{IN}^T =$	0.9418	0.9651	0.9302	0.9883	0.9440	0.9456	0.9625	0.9863
$\mathbf{RS}_L{}^T =$	1	1	1	1	1	1	1	0.3684
$\mathbf{IN}_L{}^T =$	1	1	1	1	1	1	1	0.9863
$\mathbf{RS}_G{}^T =$	0.5714	1	0.4286	1	1	1	1	1
$\mathbf{IN}_G{}^T =$	0.9418	1	0.9302	1	1	1	1	1
$\mathbf{RS}_S{}^T =$	0.5000		0.7142		0.3486		0.5177	
$\mathbf{IN}_S{}^T =$	0.9534		0.9593		0.9449		0.9724	
$\mathbf{RS}_{SL}{}^T =$	1		1		1		0.6315	
$\mathbf{IN}_{SL}{}^T =$	1		1		1		0.9863	
$\mathbf{RS}_{SG}{}^T =$	0.4286		0.5714		1		1	
$\mathbf{IN}_{SG}{}^T =$	0.9418		0.9302		1		1	

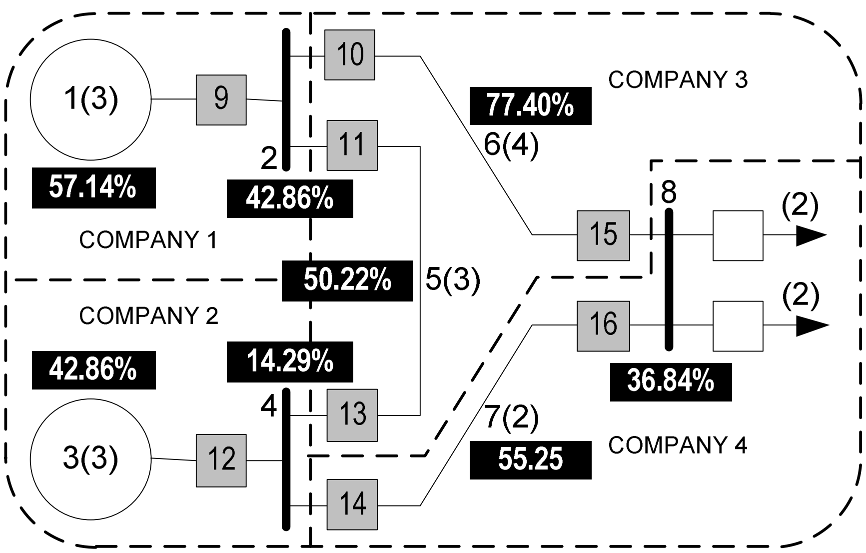


Figure 5 – Component Resilience

This example shows that the bus 4 of Company 2 is the component with the least resilience in this network, and can be immediately re-energized in only 14.29% of the forced outages it suffers, due to the low transmission capacity of the connected lines. In compensation, line 6 from Company 3 is the component with highest resilience, and can be immediately re-energized in only 77.4% of the forced outages it suffers, due to its capacity. As a consequence, companies 2 and 3, owners of these assets, present the maximum and minimum network resilience of 71.42% e 34.86% respectively. The high inoperability, over 90% of all components and connected companies result from the low connectivity among the components, and from the strong connectivity of companies 1 and 2. These factors contribute to a resilience of 48.71% for the whole network, and an inoperability of 95.99%, estimated by equations (30) e (31).

10. Conclusions

The following aspects distinguish the proposed method, in evaluating resilience and inoperability indexes of power systems, due to cascading disturbances in electrical networks:

(a) Consideration of maintenance, operation and protection of power systems;
(b) Simulation of grid topology, with forced and planned functional dependencies;
(c) Graphical representation of functional dependencies by directed graphs;
(d) Modeling of protection reliability and reach;
(e) Inclusion of remote causalities from teleprotections and from load and generation shedding;
(f) Use of traditionally available data from maintenance and operation;
(g) Elicitation of the contribution and shared responsibilities among connected companies; and
(h) Formalization by matrix algebra, with trivial implementation on computers.

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